COURSE PROPOSAL: MATH 4xy LINEAR OPTIMIZATION

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Enrollment: Elective for AM and other majors (especially CS, ECE, and BUS).

Prerequisites: MATH 332 (Matrices/ Elementary Linear Algebra).
Note this automatically implies MATH 251 (Multivariable Calculus).

Lecture Schedule: 2 75 minute lectures per week.

Assessment:

Homework/Project	25-50%
Quizzes/Tests	30-50%
Final Exam	25-40%

Textbook: D. Bertsimas and J. Tsitsiklis, Introduction to Linear Optimization, Athena Scientific, 1997.

Course Outline:

1. Introduction [2 hours]
    a. LP: formulations and examples
    b. Piecewise linear convex objective functions
    c. Graphical representation and solution
2. Geometry of Linear Programs [4 hours]
    a. Polyhedra and convex sets
    b. Extreme points, vertices, and basic feasible solutions
    c. Degeneracy of basic solutions
    d. Existence and optimality of extreme points
3. Simplex Method [7 hours]
    a. Optimality conditions
    b. Simplex method
    c. Revised simplex method and full tableau implementation
    d. Anticycling: Bland's rule
    e. Initial basic feasible solution
    f. Computational efficiency of the simplex method
4. Duality Theory and Sensitivity analysis [6 hours]
    a. Dual linear program
    b. Duality Theorems and Complementary Slackness
    c. Dual Simplex method
    d. Farkas' Lemma and its application to duality theorem
    e. Sensitivity analysis and Parametric programming

5. Large Scale Optimization [4 hours]
    a. Delayed column generation and Dantzig-Wolfe decomposition
    b. Cutting plane methods and Benders decomposition
6. Optional Topics (selected based on class composition and background) [9 hours]
    a. Interior Point Methods
        i. The von Neumann algorithm
        ii. The affine scaling algorithm
        iii. The primal path following algorithm
    b. Network Flow Problems
        i. The minimum cost flow problem and the Network simplex algorithm
        ii. The maximum flow problem and the Ford-Fulkerson algorithm
        iii. The assignment problem and the Auction algorithm
    c. Integer Programming
        i. Gomory Cuts and Cutting plane algorithms
        ii. Branch and bound
        iii. Dynamic programming
        iv. IP duality and Lagrangian Relaxation
7. Exams and Overflow [3 hours]